01-09-2025

WorldMathBook

English

For high school and beyond TRIAL PAGES

Content:

Preface, WorldMath - English

Mathematics is our most accurate science.

Mathematics is a beautiful science.

Some study mathematics alone, but most people use it as a tool for physics, biology, medicine, engineering science, economy, ……, for everything.

For high school and more. We start with the four basic arithmetic operations, and finish in the first or second semester of the study for bachelor or candidate.

The language is clear, understanding is in focus, technical terms are explained.

There is also an exercise book with problems and proposed solutions.

The book is independent of which formula collection is used.

The book is also independent of using a calculator or a calculation program.

And one more thing. Mathematics is not becoming more and more complicated as we go along. That is my personal experience, and I see it with the students too. Of course you will learn more, but the next step is not harder.

Author: Tom Pedersen, Mechanical-Processing Engineer, Ph.D. from Brunel University. I have been employed in business as a project leader and consultant, - as a researcher, and as a lecturer at technical colleges in Elsinore and Copenhagen as well as at the Danish Technical University, where I am currently employed. I have given lectures within several subjects including a lot of mathematics. I have been a lecturer within all the subjects presented in this book…….Enjoy!

Tom Pedersen, September 2025.

Percent

Percent means ”out of a hundred”, which means a fraction with 100 as the denominator.

$\frac{1}{2}$ means 1 out of 2. If we multiply by 50 in the numerator and denominator we get $\frac{50}{100}$ or 50 out of 100 or 50%. In brief:

$$\frac{50}{100} = 50\%$$

Examples

$$\frac{1}{5} = \frac{20 \cdot 1}{20 \cdot 5} = \frac{20}{100} = 20\%$$

$$\frac{1}{8} = \frac{12{,}5 \cdot 1}{12{,}5 \cdot 8} = \frac{12{,}5}{100} = 12{,}5\%$$

$$\frac{1}{4} = \frac{25}{100} = 25\%$$

and as a decimal number

$$\frac{1}{2} = \frac{50 \cdot 1}{50 \cdot 2} = \frac{50}{100} = 50\% = 0{,}5$$

$$\frac{1}{4} = \frac{25 \cdot 1}{25 \cdot 4} = \frac{25}{100} = 25\% = 0{,}25$$

$$\frac{3}{4} = \frac{25 \cdot 3}{25 \cdot 4} = \frac{75}{100} = 75\% = 0{,}75$$

$$\frac{3}{8} = \frac{12{,}5 \cdot 3}{12{,}5 \cdot 8} = \frac{37{,}5}{100} = 37{,}5\% = 0{,}375$$

Percent is out of a hundred. A decimal number is out of one.

1 is one whole. 100% is also one whole.

$$1 = \frac{100}{100} = 100\%$$

2.

Yesterday a certain dress cost 200 pounds. Today it has risen to 225 pounds. What is the rise in %?

200 pounds corresponds with 100%.

The rise is 225 - 200 = 25 pounds, which must be seen in proportion with the 200 pounds:

$\frac{25}{200} = 0.125 = 12.5\%$ which is the answer

3.

Yesterday a certain dress cost 200 pounds. Today the price has dropped to 175 pounds. What is the price reduction in %?

200 pounds corresponds with 100%.

The reduction is 200 - 175 = 25 pounds, which must be seen in proportion with the 200 pounds:

$\frac{25}{200} = 0.125 = 12.5\%$ which is the answer

The information could be given as: Today -12.5% for this dress.

4.

The price for a certain machine is 1000 pounds without VAT.

1000 pounds corresponds to 100%. Inclusive of 25% VAT the price is:

$1.25 \cdot 1000 = 1250$ pounds which is the answer

or

$100\% + 25\% = 1000 + 0.25 \cdot 1000 = 1250$ pounds

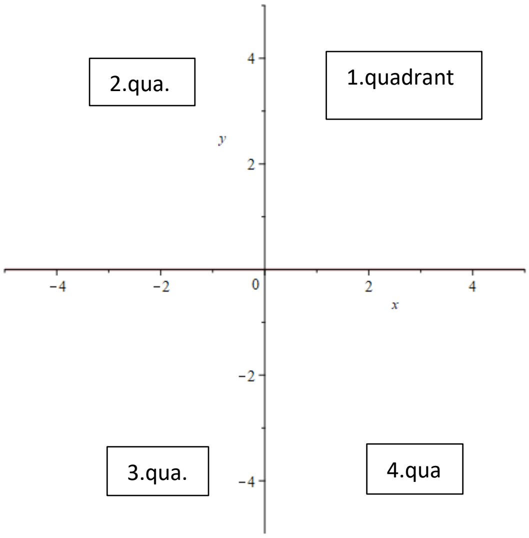
Part 2. The coordinate system in the plane (2D) and functions

The coordinate system and distance

We live in a world of three dimensions, we call it the space and it consists of length, width, and height.

If we work in two dimensions, we call it the plane, and it consists of two directions for instance horizontal and vertical. We may also call the directions for axis. Then we have the *first-axis* and the *second-axis*; or in more technical terms: The *abscissa* and the *ordinate*, both from Latin. Abscissa means ”out (ab) from here (cis)”, which may be pictured by standing at the starting point and looking horizontally at the horizon. The ordinate means the ordinary, which is vertical (all other directions would not be ordinary).

In mathematics we often use the words x-axis and y-axis,



but they can be called other things. In physics the first axis could be t for time, and the second axis could be v for velocity (velox in Latin). In economy the first axis may be months and the second axis may be costs. And so on.

The axis divide the plane in four quarters named the four quadrants. The first quadrant is where x and y are positive (both are +). Then we rotate counter clockwise to the 2. 3. and 4. quadrant.

The axis form a right angle and intersect in a common starting point, denoted like this: $(x,y) = (0,0)$. The starting point is called Origo (ancient Greek) or just O.

At the axis we chose a scale suitable for the task. Usually, we chose the same scale for the two axes, but that depends on what we are going to plot. If the scales are alike, we use the technical term: equidistant.

In all it is called a coordinate system (co(with) ordinate(the ordinary) system). It is being used everywhere.

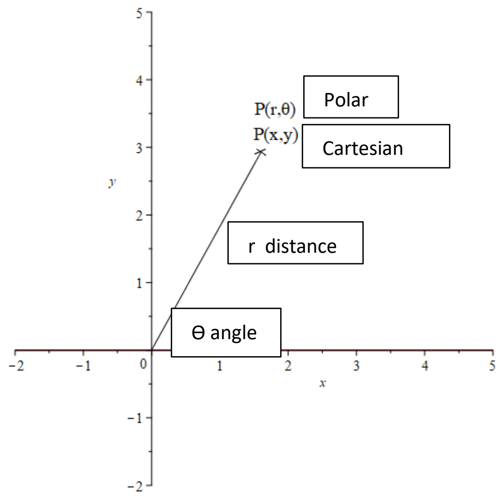
For instance, the coordinate system is used to show how a function varies: the straight line function, the parabola function, the sinus-function, and so on.

We consider x first, and y as what follows. Therefore, the x-values of a function are also called the *domain*, and the y-values are called the *range* (sometimes: the amount of value). Denotations are not commonly used because the words domain and range are brief enough and very informative, but if we call the function, f, the denotations are: Domain, D(f) - and Range, G(f). (R(f) for range would have been the logical choice, but R is used for something else).

The demand for a function is that for each x-value there is only one y-value. Therefore a function flow in a coordinate system cannot go forth and back since that would imply more y-values for one x-value. If that is needed we talk about a vector function or a parameter function which will be discussed in Part 4.

The ordinary rectangular coordinate system is also called the Cartesian coordinate system after the mathematician Descartes.

Coordinates may also be denoted by polar coordinates: (distance from Origo , angle with +x axis). See the figure:



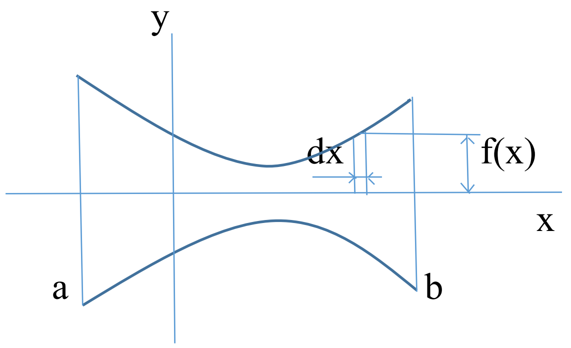
We will consider polar coordinates a little more at the end of the book.

Now it is about normal (Cartesian) coordinates.

Volumes

We can rotate a 2D area around the x or y-axis and have a 3D volume.

The formula for rotation around the x-axis derives



If we rotate our infinitesimally thin strip around the x-axis we have a micro cylinder. A macro cylinder has the volume

$V = \pi \cdot r^2 \cdot l$ l for length

for our micro cylinder the volume is

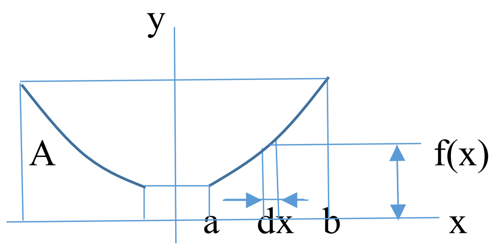
$dV = \pi \cdot f(x)^2 \cdot dx$

by integration (gathering all micro cylinders) from a to b

$V = \pi \cdot \int_a^b f(x)^2 \, dx$ the rotation volume around the x-axis

Thus, the volume can be calculated when we have an expression of the function, which informs how the radius varies.

The formula for rotation around the y-axis derives



By rotation of our infinitely thin strip around the y-axis, we have a cylinder shell with volume

$dV = \text{height} \cdot \text{circumference} \cdot \text{micro-thickness}$ =>

$dV = f(x) \cdot 2\pi x \cdot dx$ =>

and when we integrate (gather all the micro cylinder shells) from a to b, the volume - calculated numerically (x or f(x) may be negative) - is

$V = \left| 2\pi \cdot \int_a^b x \cdot f(x)\, dx \right|$ the rotation volume around the y-axis

For the figure shown the rotation volume looks like the space under the stands in a stadium.

We may also view the rotation volume as the area A rotated around the y-axis.

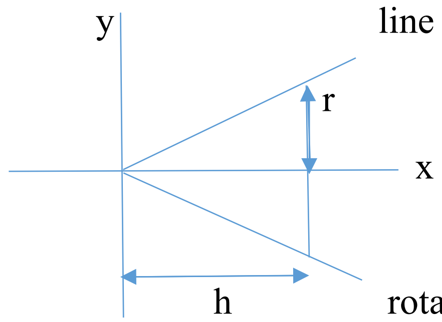
If $a = 0$ there will be no hole in the middle.

Examples

1.

We will find the formula of a cones volume.

We rotate a line segment once around the x-axis, and have a cone that lies down.



line segment: $f(x) = a \cdot x = \frac{r}{h} \cdot x$

rotated line segment

$V = \pi \cdot \int_a^b f(x)^2\, dx$ =>

$V = \pi \cdot \int_0^h (\frac{r}{h} \cdot x)^2\, dx$ r and h are constants ⇔

$V = \pi \cdot \left(\frac{r}{h}\right)^2 \cdot \int_0^h x^2\, dx$ ⇔

$V = \pi \cdot \left(\frac{r}{h}\right)^2 \cdot \left[\frac{x^3}{3}\right]_0^h$ ⇔

$$V = \pi \cdot \left(\frac{r}{h}\right)^2 \cdot \left(\frac{h^3}{3} - 0\right) \quad \Leftrightarrow$$

$$V = \frac{\pi}{3} r^2 h$$ which is the formula for a cones volume

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WorldMathBook, Exercises English

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Content:

Part 3.

Differentiation and Integration - exercises

Part 4. Vectors – exercises

Preface, WorldMath – English, exercises

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With questions and proposed answers.

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Problems

4A.052

Find both the acute angle, as well as the obtuse angle, between the planes

α: $2x - 3y + z = 8$ β: $2x + y - 4z = -8$

4A.053

Find the distance from point P(3, 7, 2) to the plane

α: $3(x-1) + 2(y+5) - 2(z-2) = 0$

2B.09

The earth has a circumference of 40 000 km along a longitude, and it is nearly spherical. We stand at the beach with our eyes 2 m above the sea level and look to the horizon. How far away is the horizon?

How far away can we see the horizon if we are 40 m, 100 m, 1000 m, above the sea?

Proposed solutions

4A.052

The angle between the planes equals the angle between their normal vectors:

$$\boldsymbol{n}_\alpha = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \quad \boldsymbol{n}_\beta = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} \quad \Rightarrow \quad |\boldsymbol{n}_1| = \sqrt{14} \quad \text{and} \quad |\boldsymbol{n}_2| = \sqrt{21}$$

Formula $\quad cos\ v = \frac{\boldsymbol{n}_\alpha \cdot \boldsymbol{n}_\beta}{|\boldsymbol{n}_\alpha| \cdot |\boldsymbol{n}_\beta|}$

Here $\quad cos\ v = \frac{-3}{\sqrt{14} \cdot \sqrt{21}}$ => $v \approx 100°$ the obtuse angle

And $\quad u \approx 180 - 100 \approx 80°$ the acute angle

4A.053

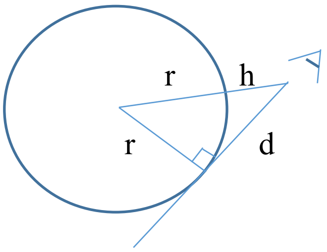
$3x - 3 + 2y + 10 - 2z + 4 = 0$ and point (3, 7, 2) =>

$$dist. = \frac{|3\cdot 3+2\cdot 7+(-2)\cdot 2+11|}{(9+4+4)^{½}} = \frac{30}{\sqrt{17}} \approx 7.28$$

2B.09

We find the radius of the earth from:

$$O = 2\pi r \quad \Leftrightarrow \quad r = \frac{O}{2\pi} = \frac{40\ 000}{2\pi} \approx 6378\ km \approx 6\ 378\ 000\ m$$



Pythagoras:

$$d^2 + r^2 = (r+h)^2 \quad \Leftrightarrow$$

$$d^2 = (r+h)^2 - r^2$$

h = 2 m:

$$d^2 = 6\ 378\ 002^2 - 6\ 378\ 000^2 \quad \Leftrightarrow \quad d = 5051\ m \approx 5\ km$$

h = 40 m:

$$d^2 = 6\ 378\ 040^2 - 6\ 378\ 000^2 \quad \Leftrightarrow \quad d = 22\ 589\ m \approx 23\ km$$

h = 100 m:

$$d^2 = 6\ 378\ 100^2 - 6\ 378\ 000^2 \quad \Leftrightarrow \quad d = 35\ 716\ m \approx 36\ km$$

h = 1000 m:

$$d^2 = 6\ 379\ 000^2 - 6\ 378\ 000^2 \quad \Leftrightarrow \quad d = 112\ 947\ m \approx 113\ km$$